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The fifth graders' mathematisation process in solving contextual problems

Hamzah Upu, Djadir & Syahrullah Asyari

Universitas Negeri Makassar
Makassar, Indonesia

ABSTRACT: Revealing the students' mathematisaton process when they are solving a given problem is essential, because when they write a numeric such as 3/4, the teacher would not know what it meant without the students' expression (communication) of the meaning. This study investigated the mathematisation process of grade five students in solving contextual problems concerning fractions, viewed from a mathematics ability difference among three groups representing high, moderate and low mathematical ability. The research results reveal differences in the horizontal and vertical mathematisation process among the three groups. The subjects' actions cannot be split directly into two major consecutive parts that is the horizontal mathematisation process that occurs wholly at first and, then, is followed by the other process. This fact is in line with de Lange's research findings.

INTRODUCTION

There are several points that researchers and scientists have identified as the causal factors of students' difficulty in learning fractions. They are: 1) *properties of the fraction itself* [1]. The fact is that a fraction comprises a series of constructs [1]. There are at least five interrelated constructs; namely, part-whole, ratio, operator, quotient and measure [2]. In addition, the written form of fractions is not easy to compare and it is difficult to line them up in order. Again, there are a lot of rules in fraction arithmetic, which are more complex than natural numbers [3]; the definition of operations utilised on fractions is still abstract [4]; 2) *learning approaches that a teacher uses to teach fraction* [1]. Teachers present fractions abstractly [4]. He/she does not familiarise students with the encountered environment in terms of fractions [4]. The teacher introduces fractions merely as part of geometrical figures [5]. In addition, teachers tend to introduce algorithms for operations concerning fractions prior to students understanding the concept [4]. This is in line with Reys, that teachers jump to conclusions to arrive at symbolisation and operation without developing a strongly conceptual foundation about numbers [6]. Further, teachers pay too much attention to the formulation and use of computation rules, whereas the fundamental concept of fractions has not yet evolved [4]; 3) *students view fractions as a part of a form or quantity and not as a number* [5]; and 4) *fractions are less familiar in daily life and not as easily described as natural number* [3].

At least three definitions of mathematisation process model have described it as a cycle; namely, the models by vom Hofe cited by Prediger [7], de Lange [8], and Murata and Kattubadi [9]. The mathematisation process described by vom Hofe et al is based on Fischbein's mental model [7]. The mental model is to explain students' difficulties in learning mathematics. Fischbein viewed this process as a meaningful interpretation about a phenomenon or concept [7].

De Lange also describes mathematisation as a cycle [8]. This mathematisation process was percived in light of Freudenthal's view that mathematics is a human activity. Therefore, according to him, mathematics must be related to reality, it must be close to student experience, and it must be relevant to society and, therefore, valuable for humans. Murata and Kattubadi describe it as process of modelling a situation mathematically that requires the student to extract information from the situation. And then, this process focuses on specific information relating to the situation, although not necessarily mathematical, for the solving process (model of the situation). Further, students develop quantitative information based on their experience to be utilised in problem-solving (model for the situation) [9].

The mathematisation process is differentiated by Treffers under two phases, that is horizontal mathematisation and vertical mathematisation [10][11]. Horizontal mathematisation is the process of solving contextual problems from the real world. In horizontal mathematisation, students try to carry out problems from the real world using their own ways and using their own language and symbols. Vertical mathematisation is a process of formalising mathematical concepts.

Within vertical mathematisation, students try to devise a general procedure that can be used to address similar problems directly without any contextual assistance.

In this study, the authors refer to the *teaching for problem-solving* approach, where learning material is delivered over earlier in order that it can be utilised within problem-solving. This approach is conducted within two phases that is delivering material and solving problems [9]. This study focused on the problem-solving aspect, which was conducted, once the teacher completed materials underpinning the contextual problem that is provided for students. Thus, problem-solving here does not merely reveal students' activities in the mathematisation process as the focus of this study, but also it plays a role as a means of formative evaluation that will recommend that the teacher improves learning quality.

Mathematics problems in school mathematics include verbal, picture and mathematics symbols or combinations of them. The verbal form can be such as stating symbols in words; giving instructions or expressing aims in finding a solution; stating a mathematics context; or *real-world* or social/cultural context, real or imagined [12]. Meanwhile, the context can be considered as a specific situation [10] or a circumstance that involves students [13].

The word *situation* here refers to the word *world*, where students live. In addition, the word *situation* can be simply analogised as a theme. De Lange classifies context into three levels based on the benefit aspect, they are: 1) first order context - this contains only translation of mathematics problems textually and explicitly; this context can be found in textbooks at school that are usually presented as word problems; 2) second order context - this gives students the opportunity to do mathematisation; within this context, problem are given to students, and they are expected to be able to find relevant mathematics concepts, organise information and, then, carry out the problem; in this context, problem position (usually such problem related to real world) is highly essential, and mathematics serves as a tool to organise reality; and 3) third order context - this context enables students to find or construct new mathematics concepts or ideas; this is the most important context in realistic mathematics education, because it satisfies characteristics for the conceptual mathematisation process [8].

In this study, a contextual problem concerning fractions is a situation of *real world* or social context, real context or at least, it can be imagined, which is related to fractions requiring appropriate action, along with the unavailability of a way to overcome the situation. Whereas the contextual problem type is used for contextual problems concerning fractions, which are personal to students, because it is closer to students' life or habituation, so that the problem given is really contextual to them. The contextual type is of the second order type, because within this level, students are enabled to go through the mathematisation process to the contextual problem given, based on their knowledge and skill concerning the topic or concept of fractions that they have learned.

Briefly, one can state that contextual problem-solving concerning fractions in this research is a process or attempt that individuals pass through when giving responses or coping with constraints using their skill and knowledge to arrive at an appropriate solution to the contextual problem concerning fractions, which is unfamiliar to students, if an answer or solution method is still puzzling.

RESEARCH METHOD

This was qualitative research, and the research objective was to investigate and describe the profile of the mathematisation process of primary school students in solving contextual problems concerning fractions viewed from mathematics ability differences. The research subjects that were selected from a larger group were three Grade 5 students at an elementary school in Pertiwi Makassar, Makassar, Indonesia. The selection was based on the Indonesian Curriculum on Educational Unit Level, because within this level, students have been provided with mixed fractions, covering addition, subtraction, multiplication and division of various fractional forms. At this level, students were also provided with a topic about the use of operations on mixed fraction in everyday life, such contextual problems, which is a familiar topic to students.

The research subjects were selected according to their mathematics ability. Initially, a number of Grade 5 students were provided with a mathematics ability test. Further, from the test results, they were organised into three groups, covering students of high, moderate and low abilities. This grouping aimed to investigate the similarity or difference of students' activities in the process of mathematising contextual problems concerning fractions for students with different ability levels. After passing through all series in the process for selecting research subjects, the researchers could then justify three students as the research subjects. This justification was administered after sorting students' learning ability test scores from the lower to the higher scores in each ability category and, then, determining the median of the scores. The median score would then establish the research subjects.

Table 1: Research subjects.

No	Name	Score (median)	Ability category
1	NLA	41	Low
2	API	69	Moderate
3	NBC	86	High

After identifying the research subjects, the research was conducted based on the mathematics ability test and the test for the profile of the mathematisation process as the main instruments.

Every morning, Syamsul goes to school on foot. When he is one-fifth of his way to school, he passes a food stall; further, when he is one-third of his way, he passes a school uniform shop; and when he is half way, he passes a bookshop. When Syamsul is in front of that bookshop, he still has to walk 50 metres to his school. What is the distance of Syamsul's house to his school? Explain your answer!

The data gathering in this research was conducted through the following stages: 1) the first step of data gathering was conducted by providing the research subjects with the test for the profile of the mathematisation process (TPMP). After the subject completed the TPPM, a task-based interview was administered. The interviews in this research were unstructured and taken directly, i.e. the interview protocol utilised here covered guidelines to problems raised. This unstructured interview was used to get information in detail and in depth from the subjects. The interview results were then recorded by digital camera in order that research data, such as the subjects' activities and utterances were preserved and saved; 2) the recordings were then transcribed and coded; 3) sometime later, the second step of data gathering was conducted by providing the research subjects with the same TPMP. The process was like in point 1 above; 4) the recordings were then transcribed and coded; 5) categorising data; 6) reducing and presenting; 7) a time triangulation was undertaken by comparing the data collected in the first step and that in the second step. If the triangulation results showed that the data in the first step were consistent, then, valid data have been obtained. However, if they showed inconsistency, then, the third step of data gathering was conducted through the same steps as previously. Furthermore, comparing the data of the first and third steps was likewise done for the second and the third steps. The comparison results showing consistency formed a reference in analysing the data to answer research questions. If the data were still inconsistent, the same process of triangulation as mentioned was continued until consistency was found in the research subjects' responses; and 8) if the triangulation results showed that the data of the first step were inconsistent, then, the i-th step of data gathering would be undertaken by providing research subjects with TPMP, $i \geq 3$. The i-th data were then compared to the (i – j)-th data, $j = 1,2, ..., (i – 1)$. This was conducted repeatedly until valid data were obtained.

In this research, data analysis was done when the data collection was taking place. This means that data analysis and data collection took place simultaneously [14][15]. Nevertheless, in order to be clear, the exposition about data collection and data analysis are presented separately.

When the researchers collected the data, analysis was conducted by: 1) making a transcription and coding; 2) categorising data; 3) reducing data; 4) presenting data; 5) interpreting the profile of mathematisation process; and 6) drawing a conclusion.

RESEARCH FINDINGS AND DISCUSSION

The three research subjects showed the mathematisation process as presented in Table 2.

Table 2: The mathematisation process of subjects S1, S2 and S3.

No	Mathematisation process	Description	Sub. S1	Sub. S2	Sub. S3
1	Horizontal	Identifying mathematics concept, which is relevant to the real-world problem			
		Representing the problem using a variety of ways, including organising problem suitable with the relevant mathematics concept, as well as formulating appropriate assumption			
		Finding a relationship between problem language and formal language of mathematics in order for the real problem to be understood mathematically			
		Finding regularity, relationship and pattern related to problem			
		Translating the problem into a mathematics form, that is in a mathematics model			
2	Vertical	Using a variety of mathematical representations			
		Using symbols, language and formal mathematics processes			
		Adapting and developing a mathematics model, as well as combining various model			
		Making mathematical argumentation			
		Generalising			

Notes: The shaded area means the activities that subjects pass; the unshaded area means the activities that subjects do not pass

Firstly, the researchers considered subject S1. Based on the results of data interpretation for the two mathematisation processes in Table 2, it is known that in solving the contextual mathematics problem given, S1 passed all activity indicators in the process of horizontal mathematisation. However, this was different from the process of vertical mathematisation, where S1 did not use various representations. Besides, S1 only used one feature of representation; namely, mathematical solution. In addition, S1 did not make a generalisation.

Secondly, subjects S2 and S3 were considered. Based on the results of data interpretation for the two mathematisation processes in Table 2, it is known that in solving contextual problem given, subject S2 and subject S3 did not pass all indicators of mathematisation processes, either horizontal or vertical. But, they showed similar mathematisation processes. They did not pass two of five activity indicators in the horizontal mathematisation process; namely, representing mathematically in different ways and attempting to find regularity, relationships and patterns related to the problem. Meanwhile, in the vertical mathematisation process, they did not pass three of the five activity indicators, namely: they did not use a variety of mathematical representations; did not adapt and develop mathematical models, as well as combining various models and did not make a generalisation.

Despite the main findings, there are some other findings. Of the three research subjects, there was something unique in subject S3, when S3 was asked about the number of fractions mentioned in the problem given. Truly, there are three fractional numbers mentioned, they are: $\frac{1}{5}$, $\frac{1}{3}$, and $\frac{1}{2}$. But, S3 stated that there are only two fractional numbers; namely, $\frac{1}{5}$ and $\frac{1}{3}$. To S3, a half was not a fraction, since if a half was a fraction, it means the fraction should be called as one over a half.

CONCLUSIONS

Based on the aforementioned research results and discussion, some conclusions may be drawn about the profile of the research subjects' mathematisation process in solving a contextual problem concerning fractions, as follows:

1. Subject with high ability in mathematics:

   The profile of the horizontal mathematisation process of the subject with high ability in mathematics is as follows: a) identifying a mathematics concept, which is relevant to a real-world problem; b) representing the problem using a variety of ways, including organising the problem with the relevant mathematics concepts, as well as formulating appropriate assumptions; c) finding a relationship between the problem language and formal language of mathematics in order for the real problem to be understood mathematically; d) finding regularity, relationships and patterns related to the problem; and e) translating the problem into mathematics form, that is in mathematics model. Whereas, activities in vertical mathematisation process are: a) not using a variety of different mathematical representations; b) using symbols, language and a formal mathematics process; c) adapting and developing mathematics models, as well as combining various models; d) making mathematical argumentation; and e) not generalising.

2. Subjects with low and moderate abilities in mathematics:

   The profile of horizontal mathematisation process of the subjects with low and moderate abilities in mathematics is as follows: a) identifying a mathematics concept, which is relevant to a real-world problem; b) not representing the problem using a variety of different ways, including not organising the problem with the relevant mathematics concepts, as well as not formulating appropriate assumptions; c) finding relationships between problem language and formal language of mathematics in order for real problem can be understood mathematically; d) not finding regularity, relationships and patterns related to problem; and e) translating problems into mathematics form, that is into a mathematics model. Activities in the vertical mathematisation process are: a) not using a variety of mathematical representations; b) not using symbols, language and formal mathematics processes; c) adapting and developing a mathematics model, as well as combining various models; d) making mathematical argumentation; and e) not generalising.

Suggestions

Based on the research results and discussions, the authors offer some suggestions as follows:

1) Mathematisation process, mathematics ability and students' mathematics experience are three indivisible aspects when solving a mathematics problem. Therefore, the teacher needs to consider all these aspects when teaching with a problem-solving approach;
2) Before giving a contextual problem concerning fractions to solve, the teacher should make students familiar with word problems about fractions with diverse complexity;
3) Teachers need to equip students with mathematics ability, such as: the use of symbols, logical reasoning, and computation, since this ability would contribute to the appropriateness of students' mathematisation results;

4) Teachers should enrich students with various *names* of a mathematics objects. For example, a half that is written as ½ may also be called as one-second or one over two. This is because the existence of students considering that a half is not a fraction, but one over two is a fraction;
5) This research used only one problem-solving question. Therefore, to subsequent researchers who would study a similar topic, it is suggested to vary questions or problems with similar or the same complexity problems as those provided to students in the early assignment or interview.

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